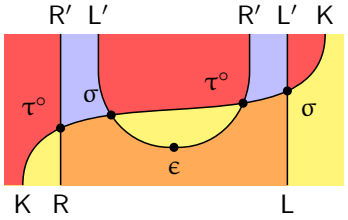
String Diagrams - Exercise Sheet 1

Exercise 1

You encounter the following string diagram in a paper. Convert it back to conventional linear notation, noting any assumptions you have to make:



Exercise 2

Assuming the following natural transformations,

$$\alpha : \mathsf{Id} \dot{\to} \mathsf{Id}, \qquad \eta : \mathsf{Id} \dot{\to} \mathsf{M},$$
$$\mu : \mathsf{M} \circ \mathsf{M} \dot{\to} \mathsf{M}, \qquad \rho : \mathsf{Id} \dot{\to} \mathsf{M} \circ \mathsf{N},$$
$$\sigma : \mathsf{Id} \dot{\to} \mathsf{L} \circ \mathsf{R}, \qquad \lambda : \mathsf{H} \circ \mathsf{T}. \to \mathsf{M} \circ \mathsf{H},$$

convert the following symbolic expressions into string diagrams:

1. $\mathsf{L} \circ \alpha \circ \alpha \circ \mathsf{R}$,
2. $\mathsf{L} \circ \alpha \circ \mathsf{R} \cdot \mathsf{L} \circ \alpha \circ \mathsf{R}$,
3. $\mu \circ \mathsf{N} \cdot \eta \circ \mathsf{M} \circ \mathsf{N} \cdot \rho$,
4. $\mu \circ \mathsf{N} \cdot \mathsf{M} \circ \rho \cdot \eta$,
5. $\mathsf{L} \circ \mathsf{M} \circ \sigma \circ \mathsf{N} \circ \mathsf{R} \cdot \mathsf{L} \circ \rho \circ \mathsf{R} \cdot \sigma$,
6. $\mathsf{L} \circ (\mathsf{M} \circ \sigma \circ \mathsf{N} \cdot \rho) \circ \mathsf{R} \cdot \sigma$,
7. $\mu \circ \mathsf{H} \cdot \mathsf{M} \circ \lambda \cdot \lambda \circ \mathsf{T}$.

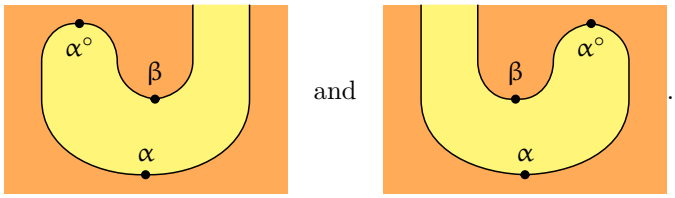
Try to make the diagrams as colorful as possible.

Exercise 3

Part I

For a pair of functors $\mathsf{F} : \mathcal{C} \to \mathcal{D}$ and $\mathsf{G} : \mathcal{D} \to \mathcal{C}$ and natural isomorphisms $\alpha : \mathsf{G} \circ \mathsf{F} \dot{\to} \mathsf{Id}$ and $\beta : \mathsf{F} \circ \mathsf{G} \dot{\to} \mathsf{Id}$ we can form two "hockey stick"-shaped diagrams

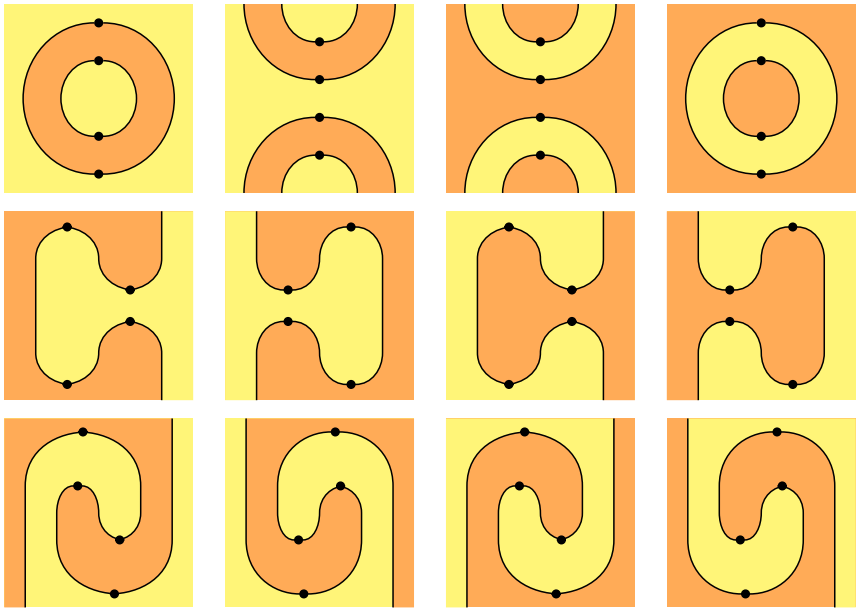
as follows:



Prove that these two composites are equal.

Part II

With α and β as in the previous part, show that each diagram below, containing exactly one copy of α, β, α° and β° is equal to the identity.



Exercise 4

We encountered the gray regions for the terminal category in lectures. Would it ever be useful to have the gray region appear anywhere except at the the right hand side of a string diagram? Justify your answer.